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Abstract

This paper studies dichotomous majority voting in common interest committees where each member receives not only a private signal but also a public signal observed by all of them. The public signal represents, e.g. expert information presented to an entire committee and its quality is higher than that of each individual private signal. We identify two informative symmetric strategy equilibria, namely i) the mixed strategy equilibrium where each member randomizes between following the private and public signals should they disagree; and ii) the pure strategy equilibrium where they follow the public signal for certain. The former outperforms the latter. The presence of the public signal precludes the equilibrium where every member follows their own signal, which is an equilibrium in the absence of the public signal. The mixed strategy equilibrium in the presence of the public signal outperforms the sincere voting equilibrium without the public signal, but the latter may be more efficient than the pure strategy equilibrium in the presence of the public signal. We suggest that whether expert information improves committee decision making depends on equilibrium selection.

Keywords: information aggregation, strategic voting, expert information
JEL Classification: D72, D82, D83

1 Introduction

When collective decisions are made through voting, typically each voter has not only private information known solely to herself but also "public" information observed by all voters. Examples of commonly held information in collective decision making include an "expert" opinion solicited by a committee, shared knowledge in a board that has emerged from pre-voting deliberation, and evidence presented to a jury. Such information may well be superior to the private information each voter has, in which case it would be natural to expect that voting behaviour would reflect the public information at least to some extent. Indeed, the primary

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reason for bringing shared information to a decision making body would be to improve the quality of its decision.

Meanwhile, expert information is rarely perfect, and in particular expert opinions are often alleged to have excessive influence on decision making. For example, recently IMF's advice to the governments of some highly indebted countries indeed have heavily influenced their parliamentary or cabinet decisions for austerity. However, IMF's expertise has been questioned by specialists in monetary policy, and it has been reported that IMF itself has admitted that they may have underestimated the impact of their austerity measure in Greece.[1] Financial deregulation in the 1990s seem to have been prompted by experts endorsing them, but some politicians reflect that in retrospect they may have followed expert opinions too naively.[2] Related phenomena have also been studied in social psychology. In ambiguous or uncertain situations people who ask experts for help often "internalize" the information the experts have, by believing that it is true even if they are aware that experts may make mistakes (Hogg and Vaughan, 2010). How would collective decision making through voting be influenced by shared information? If commonly observed expert information is better than the information each voter has, would the presence of such expert information improve the quality of the collective decision? Can expert information have "too much" influence?

This paper addresses these questions by introducing a public signal to a Condorcet jury model with strategic voters. The public signal is observed by all voters and superior to the private signal each voter receives, which represents e.g., expert information. As is well-known, in majoritarian dichotomous decision making without a public signal, sincere voting is a Bayesian Nash equilibrium and the Condorcet Jury Theorem (CJT) holds (Austen-Smith and Banks, 1996). That is, as the size of the decision making body becomes larger, the majority decision is more likely to be correct. It is also known that, given sincere voting, the efficiency of a Condorcet jury is negatively associated with the correlation of private signals (e.g. Ladha, 1992). In our setup, the information each strategic voter receives has both perfectly correlated component (public signal/expert information) and independent component (private signal), and thus the voting equilibria and their efficiency relative to the standard Condorcet jury without expert information are non-trivial.

We demonstrate that there are two "informative" symmetric strategy equilibria in our setup: one in which every voter follows the public signal and ignores the private signal, and the other in which they randomize between the two decisions should the private and public signals disagree. Interestingly, while the voters may ignore their private information completely, they cannot ignore the expert information completely in equilibrium. In other words, voting according only to their private signal is not an equilibrium, since if a voter knows that all the others will follow their private signals, he deviates and follows the public signal, which is by assumption

[1] `http://www.guardian.co.uk/business/2013/jun/05/imf-admit-mistakes-greek-crisis-austerity`
[2] `http://www.bbc.co.uk/news/business-13032013`

superior to his private signal.

In the mixed strategy equilibrium, the probability that each voter follows the public signal when it disagrees with the private signal can be interpreted as the "weight" the voters collectively assign on the public signal. If both signals coincide, then they vote accordingly without randomization. A remarkable feature of this equilibrium is that the weight on the public signal is optimal. That is, a welfare-maximizing social planner would also assign the same mixing probability if she could determine the voters' strategies. This readily implies that, if the voters can coordinate to play the mixed strategy equilibrium, expert information always improve welfare, despite the correlation of information it introduces. In other words, the mixed strategy equilibrium in the presence of expert information outperforms the sincere voting equilibrium in the absence of expert information.

On the other hand, the equilibrium where the voters follow expert information, which is arguably simpler than the mixed strategy equilibrium, can be less efficient than the sincere voting equilibrium in the absence of expert information. This is because, while the efficiency of the former equilibrium is "capped" by the quality of the expert information, the latter equilibrium has no such bound as the number of voters is larger, thanks to the CJT. Therefore the introduction of expert information may reduce efficiency, if the voters are to follow the expert without randomizing appropriately. Therefore our model points to practical policy questions: i) whether the voters can coordinate to play the mixed strategy equilibrium to optimally incorporate expert information collectively through randomization; and ii) if they fail to do so, whether the welfare can be higher in the absence of expert information. In relation to the examples we saw earlier, the equilibrium predictions of our models are consistent with both efficient use of expert information and excessive reliance on it, which begs the empirical question as to which equilibrium should be more plausible.

In their seminal paper Austen-Smith and Banks (1996) first introduced equilibrium analysis to the CJT with independent private signals. They demonstrated that, while in majority rule sincere voting is an equilibrium, this is not the case for super-majority rules. McLennan (1998) and Wit (1998) derived mixed strategy equilibria in the model of Austen-Smith and Banks (1996) and showed that the CJT holds in equilibrium for majority and super-majority rules. The present paper focuses on majority voting, but introduces a public signal to the analysis, which better reflects the informational environment of typical committee decision making in reality. Also we can address the question of whether one should give expert information to a committee. Note that the CJT holds through sincere voting even in the absence of expert information, which implies that as long as the committee size is large the equilibrium outcome will be efficient. In particular, our analysis indicates that an "informative" equilibrium in the presence of expert information can be less efficient than the informative equilibrium in the absence of expert information. In other words, we point to the possibility that a committee might be better off without expert information.

Expert information in our setup can be considered as information that is (perfectly) correlated across voters. Ladha (1992), Ladha (1995) and Berg (1993) studied correlated signals in majoritarian decision making with the assumption that every voter votes sincerely, and found that the more correlated the signals are, the less efficient the decision will be. Our model is different from theirs in that the voters are fully strategic and the correlation of information takes a specific form which consists of privately observed independent signals and a publicly observed signal.

The literature on deliberation in voting has studied public information endogenously generated by voters sharing their otherwise private information (e.g. Coughlan, 2000; Austen-Smith and Feddersen, 2005). In these models once a voter reveals his information credibly he has no private information. In our model public information is exogenous for the voters and every voter keeps their private information. Note that, while public information in our model can be most naturally taken as "expert information", it would also be possible to interpret the public information in our model as superior information generated through deliberation (such as "consensus" that has emerged from discussion), while a certain amount of private information remains uncommunicated for various reasons. The advantage of keeping the public information exogenous is that we can elucidate the interplay between the accuracy of the public signal and that of each private signal.

The rest of this paper is organized as follows. The next section presents our model, and we derive its equilibria in Section 3. Section 4 concludes.

2 Model

Consider a committee that consists of an odd number of agents $n \in N = \{1, 2, .., n\}$. Each agent simultaneously casts a costless binary vote, denoted by $x_i = \{A, B\}$, for a collective decision $y \in Y = \{A, B\}$. The committee decision is determined by the majority rule. The binary state of the world is denoted by $s \in S = \{A, B\}$, where both events are ex ante equally likely $\Pr[s = A] = \Pr[s = B] = \frac{1}{2}$. The members have identical preferences $u_i : Y \times S \to \mathbb{R}$ and the payoffs are normalized without loss of generality at 0 or 1. Specifically we denote the vNM payoff by $u_i(y, s)$ and assume $u_i(A, A) = u_i(B, B) = 1$ and $u_i(A, B) = u_i(B, A) = 0$, $\forall i \in N$. This implies that the agents would like the decision to be "matched" with the state.

Before they vote, each agent receives two signals. One is a private signal about the state $\sigma_i \in K = \{A, B\}$, for which the probability of the signal and the state being matched is given by $\Pr[\sigma_i = A \mid s = A] = \Pr[\sigma_i = B \mid s = B] = p$, where $p \in (1/2, 1]$. We also have $\Pr[\sigma_i = A \mid s = B] = \Pr[\sigma_i = B \mid s = A] = 1 - p$.

In addition to the private signal, all agents observe a common public signal $\sigma_E \in L = \{A, B\}$, which is assumed superior than each agent's individual signal. Specifically, we assume $\Pr[\sigma_E = A \mid s = A] = \Pr[\sigma_E = B \mid s = B] = q$ and $\Pr[\sigma_E = A \mid s = B] = \Pr[\sigma_E = B \mid s =$

$A] = 1 - q$, where $q > p$.

The public signal has natural interpretations. It can be thought of as expert information given to the entire committee as in, e.g. congressional hearings. Briefing materials presented to and shared in the committee would also have the same feature. Alternatively, it may capture shared knowledge held by all agents as a result of pre-voting deliberation. In that case, the private signal represents any remaining uncommunicated information of each agent, which is individually inferior to shared information.

The timing of our voting game is summarized as follows:

1. Nature determines the state of the world;

2. Each agent observes private and public signals;

3. Each agent votes for a decision;

4. Majority decision is implemented and payoffs are realized.

Needless to say, in the absence of the public signal, the model is identical to the classic Condorcet Jury model with majority rule where there exists a sincere voting equilibrium such that $x_i = \sigma_i$ for any i and the Condorcet Jury Theorem holds. In what follows we study Bayesian Nash equilibria of the game in which the agents also share public information. Before doing so let us define some key concepts.

Let $v_i : K \times L \to [0, 1]$ denote the probability of an agent voting for what her private signal $\sigma_i \in K = \{A, B\}$ indicates, given the public signal $\sigma_E \in L = \{A, B\}$. We restrict our attention to symmetric strategies, where every agent's conditional distribution of their vote is identical.

Definition 1. A voting strategy v_i is *symmetric* if $v_i = v$, $\forall i \in N$.

Since each agent receives two signals in the game, we formalize three classes strategies, namely i) one where v_i depends only on the private signal; ii) one where $v_i$depends on both the private and public signals; and the other where v_i depends only on the public signal.

Definition 2. A voting strategy v_i is *individually informative* if $v_i(\sigma_i, \sigma_E) = 1$, $\forall \sigma_i \in K, \sigma_E \in L$.

An individually informative strategy is analogous to the informative (or "sincere") voting in the standard voting literature with private information, where an agent votes for what the private signal indicates. Since we have two signals (private and public) for a binary decision that can potentially disagree, we introduce the notion of responding to both signals as follows:

Definition 3. A voting strategy v_i is *dually responsive* if $v_i(A, \sigma_E) \neq v_i(B, \sigma_E)$ $\forall \sigma_E \in L$, and at least $v_i(A, B) \in (0, 1)$ or $v_i(B, A) \in (0, 1)$.

When a strategy is dually responsive and both signals disagree, the agent follows neither of them with probability 1. Meanwhile there is another type of pure strategy where the agent reacts only to the public signal.

Definition 4. A voting strategy v_i is *obedient* if $v_i(A,B) = v_i(B,A) = 0$ and $V_i(A,A) = v_i(B,B) = 1$.

An obedient strategy is the pure strategy where every agent votes for what the public signal indicates with probability 1, regardless of their private signal.

As in the literature on voting with strategic agents, each agent's optimal action depends on the comparison of his expected payoffs in the event he is pivotal. The probability of agent i being pivotal is denoted by $piv(v_{-i})$:

Definition 5. The event $piv(v_{-i})$ is the event where an agent i is pivotal using strategy v_i, while others are using v_{-i}, $i \in N$.

Throughout this paper we study the equilibrium outcome of the voting game with fully rational agents, and the solution concept we use is Bayesian Nash equilibrium:

Definition 6. A Bayesian Nash equilibrium of the game is a strategy profile v^*, such that

$$E[u_i|v_i^*, piv(v_{-i}^*), \sigma_i, \sigma_E] \geq E[u_i|v_i, piv(v_{-i}^*), \sigma_i, \sigma_E], i \in N, v_i \in X \times S, \sigma_i \in K, \sigma_E \in L. \quad (1)$$

The efficiency of the committee decision making is measured with regard to the efficiency under sincere voting in the absence of expert information.

Definition 7. Suppose each of n agents receives a private signal (with accuracy $p > 1/2$) only and votes sincerely. The probability that the majority decision matches the state is denoted by

$$P_C(p,n) \equiv \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} p^k (1-p)^{n-k}.$$

Needless to say the Condorcet Jury Theory for majority voting states that $P_C(p,n) \to 1$ as $n \to \infty$. In the absence of a public signal, sincere voting is also a Bayesian Nash equilibrium (Austen-Smith and Banks, 1996).

3 Equilibria

In this section we study how the coexistence of private and public signals affects equilibrium voting behaviour, focusing on symmetric strategies. But let us first note that, as in most models in the voting literature, there are "uninformative" equilibria where all agents vote for a decision regardless of the signals and the outcome is deterministic. This holds true because no individual agent can be pivotal and hence they never influence the outcome. In what follows we consider the equilibria in which voting behaviour and the outcome depend on the signals the agents observe.

Let us first establish that the presence of expert information upsets the individually informative equilibrium, where every agent votes according to his own signal only.

Proposition 1. *Individually informative voting is not a Bayesian Nash equilibrium.*

Proof. Consider agent i's strategy in the putative equilibrium where all the other agents adopt the individually informative strategy. She computes the difference in the expected payoff between voting for A and B, conditional on her private and public signals, in the event that she is pivotal. The payoff difference is given by

$$\begin{aligned} w(\sigma_i, \sigma_E) &\equiv E[u_i(A, s) - u_i(B, s)|piv(v_{-i}), \sigma_i, \sigma_E]\Pr[piv(v_{-i}), \sigma_i, \sigma_E] \\ &= \frac{1}{2}\Pr[\sigma_E|s = A]\Pr[\sigma_i|s = A]\Pr[piv(v_{-i})|s = A] \\ &- \frac{1}{2}\Pr[\sigma_E|s = B]\Pr[\sigma_i|s = B]\Pr[piv(v_{-i})|s = B], \end{aligned} \tag{2}$$

equilibrium where the equality follows from the independence of the signals in individually informative voting. Without loss of generality, let us assume $\sigma_i = B$ and $\sigma_E = A$. From (2) we have T

$$\begin{aligned} w(B, A) &= \frac{1}{2}\left(q(1-p)\frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}}\right) \\ &- \frac{1}{2}\left((1-q)p\frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}}\right) \\ &= \frac{1}{2}(q-p)\frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}} > 0. \end{aligned}$$

The inequality holds since $q > p$. This implies that agent i votes for A despite her private signal B. Thus individually informative voting is not a Bayesian Nash equilibrium. □

The proposition has a simple intuition. Suppose that an agent is pivotal and the private and public signals disagree. In that event, the posterior of the agent is such that the votes of the other agents, who vote individually informatively, are collectively uninformative, since there are equal numbers of the votes for both A and B. Given this, the agent compares the two signals and chooses to follow the public one as it has higher accuracy ($q > p$), but the voting behaviour breaks the individually informative equilibrium in symmetric strategies.

On the other hand, it is easy to see that there exists an equilibrium where every agent votes according the public signal and ignores their own:

Proposition 2. *Obedient voting is a Bayesian Nash equilibrium.*

Proof. Consider agent i. If all the other agents vote according to the public signal, she is indifferent with respect to her vote. Thus every agent voting according to the public signal is an equilibrium. □

The reasoning is similar to the one for the uninformative equilibria where all agents vote for the same decision regardless of the signals and the probability of the majority decision matching the correct state is 1/2. However, in the obeident equilibrium the outcome does reflect one of the signals and thus is not completely uninformative. The equilibrium clearly outperforms the uninformative equilibria since $q > 1/2$. The same line of reasoning also leads to the following remark:

Remark 1. There exists an equilibrium where every agent votes against the public signal.

This equilibrium however is implausible, since from $1 - q < 1/2$ it is outperformed even by the uninformative equilibria. In what follows we rule out this equilibrium.

Later we show that there exists a mixed strategy equilibrium where both private and public signals are taken into account, and study its properties. Before deriving the equilibrium, it is useful to show that the mixed strategy equilibrium takes a "hybrid" form, where mixing occurs only when the private and public signals disagree.

Lemma 1. *Suppose there exists a symmetric equilibrium in mixed strategies. In such an equilibrium, any agent whose private signal coincides with the public signal votes according to the signals with probability* 1.

Proof. First, note that because of the symmetry of the model with respect to A and B, we can consider the case of$\sigma_E = A$ that of $\sigma_E = B$ as two independent and essentially identical games, where only the labelling of the signal and decision differs. Thus we let $v_i(A, A) = v_i(B, B) = \alpha$ and $v_i(B, A) = v_i(A, B) = \beta$. Without loss of generality, let us assume $\sigma_E = A$ to prove the lemma.

Define

$$F(A) \equiv \Pr[piv(v_{-i})|s = A] = \sum_{k=0}^{n-1} \sum_{j=0}^{\min\left(k, \frac{n-1}{2}\right)} \binom{n-1}{k} p^k (1-p)^{n-1-k} \times \binom{k}{j} \alpha^j (1-\alpha)^{k-j} \binom{n-1-k}{\frac{n-1}{2} - j} (1-\beta)^{\frac{n-1}{2}-j} \beta^{\frac{n-1}{2}-k+j} \tag{3}$$

and

$$F(B) \equiv \Pr[piv(v_{-i})|s = B] = \sum_{k=0}^{n-1} \sum_{j=0}^{\min\left(k, \frac{n-1}{2}\right)} \binom{n-1}{k} p^k (1-p)^{n-1-k} \times \binom{k}{j} \beta^j (1-\beta)^{k-j} \binom{n-1-k}{\frac{n-1}{2} - j} (1-\alpha)^{\frac{n-1}{2}-j} \alpha^{\frac{n-1}{2}-k+j}. \tag{4}$$

Using $F(A)$ and $F(B)$, we rewrite

$$w(A,A) = \frac{1}{2}\left[qpF(A) - (1-q)(1-p)F(B)\right] \tag{5}$$
$$w(B,A) = \frac{1}{2}\left[q(1-p)F(A) - (1-q)pF(B)\right]. \tag{6}$$

Note that (5) and (6) incorporate each agent's Bayesian updating on the state and the private signals other agents may have received, conditional on his own signal and the public signal.

In order to have fully mixing equilibrium, namely $\alpha^* \in (0,1)$ and $\beta^* \in (0,1)$, we must have $w(A,A) = 0$ and $w(B,A) = 0$ simultaneously for indifference. In what follows, we show that $w(A,A) > 0$ for any α and β, which implies in equilibrium we must have $\alpha^* = 1$ and if mixing occurs it must be only for β, that is, when the private and public signals disagree. Specifically, we show that $F(A) > F(B)$, which readily implies $w(A,A) > 0$ from (5).

From (5) and (6) we have $F(A) - F(B) > 0$ if

$$\begin{aligned}\alpha^j(1-\alpha)^{k-j}(1-\beta)^{\frac{n-1}{2}-j}\beta^{\frac{n-1}{2}-k+j} &> \beta^j(1-\beta)^{k-j}(1-\alpha)^{\frac{n-1}{2}-j}\alpha^{\frac{n-1}{2}-k+j} \\ \Leftrightarrow \beta(1-\beta) &> \alpha(1-\alpha) \\ \Leftrightarrow (\alpha+\beta-1)(\alpha-\beta) &> 0. \end{aligned} \tag{7}$$

To see that (7) holds we will show that in equilibrium $\alpha^* + \beta^* - 1 > 0$ and $\alpha^* - \beta^* > 0$.

Let us first observe that $\alpha^* + \beta^* - 1 > 0$. The difference in the difference in payoffs between voting for A and B is given by

$$w(A,A) - w(B,A) = \frac{q(2p-1)}{2}F(A) + \frac{(1-q)(2p-1)}{2}F(B) > 0, \tag{8}$$

since both terms in the right hand side are positive since $p, q > 1/2$. Thus, given $\sigma_E = A$, the equilibrium probability of voting for A when $\sigma_i = A$ must be strictly greater than that of voting for A when $\sigma_i = B$, which implies[3]

$$\alpha^* + \beta^* - 1 > 0. \tag{9}$$

Second, let us show that $\alpha^* > \beta^*$. We assume instead that $\alpha^* \leq \beta^*$ in equilibrium and derives a contradiction. There is no hybrid equilibrium such that $\alpha^* \in (0,1)$ and $\beta^* = 1$, because from (7) and (9), $\alpha^* \leq \beta^*$ implies $F(A) \leq F(B)$ and we may have a fully mixed equilibrium, in which case $w(A,A) = w(B,A) = 0$. From (5) we have

$$w(A,A) = 0 \Rightarrow \frac{F(A)}{F(B)} = \frac{(1-q)(1-p)}{qp}, \tag{10}$$

[3]See Lemma 1 in Wit (1998) for a similar argument.

and from (6)

$$w(B,A) = 0 \Rightarrow \frac{F(A)}{F(B)} = \frac{(1-q)p}{q(1-p)}. \tag{11}$$

We can see that (10) and (11) hold simultaneously if and only if $p = 1/2$, which is a contradiction, since $p \in (1/2, 1]$. Thus we conclude that $\alpha^* > \beta^*$ in any mixed strategy equilibrium equilibrium.

Combining $\alpha^* > \beta^*$ and (9), we can see that (7) holds. Thus we have $F(A) - F(B) > 0$ and $w(A,A) > 0$, which implies any mixed strategy equilibrium has to have a hybrid form, such that $\alpha^* = 1$. □

Lemma 1 is not surprising, in that when both signals coincide they would jointly convince an agent to vote accordingly when she is pivotal. The non-trivial part of the lemma is that this intuition holds regardless of the mixing probability when the signals disagree. Thanks to the lemma we can focus on mixing when the private and public signals disagree.

Proposition 3. *If $q \in (p, \bar{q}]$ there exists a unique dually responsive Bayesian Nash Equilibrium, where*

$$\bar{q} = \frac{\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}{1 + \left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}.$$

In the equilibrium, the agents whose private signal coincides with the public signal vote according to them with probability 1*. The agents whose two signals disagree vote according to their private signal with probability*

$$\beta^* = \frac{1 - A(p,q,n)}{p - A(p,q,n)(1-p)}, \text{ where } A(p,q,n) = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}}.$$

If $q > \bar{q}$ there is no dually responsive equilibrium.

Proof. From Lemma 1 any mixed strategy equilibrium involves $v_i(A,A) = v_i(B,B) = 1$ and $v_i(A,B) = v_i(B,A) = \beta \in (0,1)$ for any $i \in N$.

When the state and the public signal match, the probability of each individual agent voting correctly for the state is given by

$$r_A \equiv p + (1-p)(1-\beta), \tag{12}$$

and when the state and the public signal disagree, the probability of each individual agent voting correctly is

$$r_B \equiv (1-p) \times 0 + p\beta = p\beta. \tag{13}$$

To have $\beta^* \in (0,1)$, we need any voter to be indifference when the two signals disagree:

$$w(B,A) = q(1-p)\binom{n-1}{\frac{n-1}{2}} r_A^{\frac{n-1}{2}}(1-r_A)^{\frac{n-1}{2}} - (1-q)p\binom{n-1}{\frac{n-1}{2}} r_B^{\frac{n-1}{2}}(1-r_B)^{\frac{n-1}{2}} = 0 \quad (14)$$

$$\Rightarrow \frac{1-p\beta}{1-\beta(1-p)} = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}} \quad (15)$$

$$\Rightarrow \beta^* = \frac{1-A(p,q,n)}{p-A(p,q,n)(1-p)}, \quad (16)$$

such that $A(p,q,n) = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}}$. Thus when $\beta^* \in (0,1)$ we obtain a mixed strategy equilibrium of the hybrid form ($\alpha^* = 1$).

Finally, solving $\beta^* = 0$ for q, we see that $\beta^* \in (0,1)$ if and only if $q \in \left(p, \frac{\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}{1+\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}\right)$. The uniqueness follows from the fact that the left hand side of (15) is strictly decreasing in β. □

Note that there exists a mixed strategy equilibrium when the accuracy of the public signal is relatively close to that of private signal. When this is the case, there are two responsive equilibria, namely i) the obedient equilibrium where all agents follow the public signal; and ii) the dually responsive equilibrium in which they take into account both private and public signals by mixing. Meanwhile, when the public signal is sufficiently accurate relative to private signals, then the only responsive equilibrium is obedient.

Later we will show that $\bar{q}$ is strictly larger than P_C, the accuracy of majority decision in the absence of a public signal (Definition 7). In other words, if the accuracy of expert information is the same as what the agents can collectively achieve without such information, then they still incorporate both their private signals and the public signal into their decision through randomization.

The equilibrium mixing probability β^* captures the weight the agents put on the private signal relative to the public signal when the two signals disagree.

Corollary 1. *$\partial\beta^*/\partial p > 0$ and $\partial\beta^*/\partial q < 0$. That is, the more informative the private signal becomes relative to the public signal, the more weight the agents put on the private signal in the mixing probability.*

This indicates that the equilibrium behaviour fine-tunes the weights on the two signals according to their accuracy. How does the mixing probability change according to the committee size? Interestingly, as the committee size becomes larger, the weight on the private signal also decreases, and becomes zero as n tends to infinity:

Corollary 2. *$\partial\beta^*/\partial n > 0$, and $\beta^* \to 1$ as $n \to \infty$. That is, the equilibrium probability β^* of voting against the public signal when the two signals disagree is increasing in the committee* *size n. As the committee size tends to infinity the dually responsive equilibrium converges to the individually informative equilibrium, where the public signal is ignored.*

Corollary 2 can be interpreted intuitively as follows. Given the accuracies of the private and public signals, as the committee size becomes larger, the collective accuracy of private signals increases through the logic of CJT. This implies that the "value-added" of the public signal relative to each private signal ($q > p$) decreases.

Corollary 3. $\partial\bar{q}/\partial n > 0$. *That is, as the committee size becomes larger, the dually responsive equilibrium exists for a wider range of q.*

This corollary has a similar intuition to that of Corollary 2. Since the collective accuracy of private signals increases as n becomes large, in order for the agents to disregard completely, the public signal has to be much more accurate.

So far we have observed the properties of voting behaviour in the dually responsive equilibrium identified in Proposition 3. It remains to examine the efficiency of the equilibrium, with respect to the obedient equilibrium, which is the other responsive symmetric equilibrium. This is a non-trivial question to ask, not least because the public signal introduces a type of "correlation" to the information the agents receives, and it is known that correlation of private signals leads to less efficiency. As the following proposition demonstrates, however, in the dually responsive equilibrium the agents optimally take into account the public signal by mixing. In other words, if a welfare maximizing social planner were to choose α and β to maximize the probability that the majority decision matches the true state, which we denote by $P(\alpha, \beta)$, then they coincide with the equilibrium α^* and β^*.

Proposition 4. *The dually responsive equilibrium maximizes the expected welfare with respect to α and β.*

Proof. We maximize the probability of the majority outcome matching the correct state by choosing $\alpha = v_i(A, A) = v_i(B, B)$ and $\beta = v_i(B, A) = v_i(A, B)$. Conditional on the state $s = A$ and $\sigma_E = A$, let the ex ante probability of each agent voting for A be, from (12), $r_A \equiv p\alpha + (1-p)(1-\beta)$. Also from (13), conditional on the state $s = A$ and $\sigma_E = B$, let the probability of each agent voting for A be $r_B \equiv p\beta + (1-p)(1-\alpha)$. Using r_A and r_B, the ex ante probability $P(\alpha,\beta)$ that the majority decision matches the state can be written as

$$\begin{aligned}
P(\alpha,\beta) =& \Pr[M=s|s] = \Pr[M=A|s=A]P[A] + \Pr[M=B|s=B]P[B] \\
=& \Pr[M=A|s=A]\frac{1}{2} + \Pr[M=B|s=B]\frac{1}{2} = \Pr[M=A|s=A] \\
=& \Pr[\sigma_E=A|s=A]\Pr[M=A,\sigma_E=A|s=A] \\
& + \Pr[\sigma_E=B|s=A]\Pr[M=A,\sigma_E=B|s=A] \\
=& q\sum_{k=\frac{n+1}{2}}^{n}\binom{n}{k}r_A^k(1-r_A)^{n-k} + (1-q)\sum_{k=\frac{n+1}{2}}^{n}\binom{n}{k}r_B^k(1-r_B)^{n-k}.
\end{aligned} \tag{17}$$

Note that for

$$g(x) \equiv \sum_{k=\frac{n+1}{2}}^{n}\binom{n}{k}x^k(1-x)^{n-k}$$

we have

$$\frac{dg(x)}{dx} = n\binom{n-1}{\frac{n-1}{2}}\left(x(1-x)\right)^{\frac{n-1}{2}}.$$

Partially differentiating (17) with respect to α and β, we obtain

$$\begin{aligned}
\frac{\partial P(\alpha,\beta)}{\partial\alpha} =& npq\binom{n-1}{\frac{n-1}{2}}\left(r_A(1-r_A)\right)^{\frac{n-1}{2}} \\
& - n(1-p)(1-q)\binom{n-1}{\frac{n-1}{2}}\left(r_B(1-r_B)\right)^{\frac{n-1}{2}}
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
\frac{\partial P(\alpha,\beta)}{\partial\beta} =& -(1-p)nq\binom{n-1}{\frac{n-1}{2}}\left(r_A(1-r_A)\right)^{\frac{n-1}{2}} \\
& + pn(1-q)\binom{n-1}{\frac{n-1}{2}}\left(r_B(1-r_B)\right)^{\frac{n-1}{2}}.
\end{aligned} \tag{19}$$

From (19), taking the first order condition with respect β we have

$$\frac{\partial P(\alpha,\beta)}{\partial\beta} = 0 \Leftrightarrow \left(\frac{r_B(1-r_B)}{r_A(1-r_A)}\right)^{\frac{n-1}{2}} = \frac{q(1-p)}{(1-q)p}. \tag{20}$$

If (20) holds, then the derivative with respect to α, (18), is strictly positive for any $\alpha \in [0,1]$

since

$$\begin{aligned}\frac{\partial P(\alpha,\beta)}{\partial\alpha} > 0 &\Leftrightarrow \frac{qp}{(1-q)(1-p)} > \left(\frac{r_B(1-r_B)}{r_A(1-r_A)}\right)^{\frac{n-1}{2}} \\ &\Leftrightarrow \frac{qp}{(1-q)(1-p)} > \frac{q(1-p)}{(1-q)p} \\ &\Leftrightarrow p > \frac{1}{2}.\end{aligned}$$

Therefore we have a unique corner solution for α, namely $\alpha = 1$, which coincides with the equilibrium α^* in the hybrid mixed strategy identified in Proposition 3. Note that the first order condition (19) and the indifference condition for the mixed strategy equilibrium (14) also coincide. Thus $\beta = \beta^*$ satisfies the first order condition.

It remains to show that the second order condition for the maximization with respect to β is satisfied. Since $P(\alpha,\beta)$ is a polynomial it suffices to show that

$$\begin{aligned}\frac{\partial^2 P(\alpha,\beta)}{\partial\beta^2} < 0 \Rightarrow &-(1-p)nq\binom{n-1}{\frac{n-1}{2}}(r_A(1-r_A))^{\frac{n-3}{2}}(1-p-2\beta(1-p)^2) \\ &< pn(1-q)\binom{n-1}{\frac{n-1}{2}}(r_B(1-r_B))^{\frac{n-3}{2}}(p-2\beta p^2). \qquad (21)\end{aligned}$$

At $\beta = \beta^*$, (21) reduces to

$$(1-p\beta)(1-2(1-p)\beta) > (1-2p\beta)(1-(1-p)\beta),$$

which holds since $p > \frac{1}{2}$. Since $P(\alpha,\beta)$ is a continuously differentiable function on a closed interval, the local maximum at $\{\alpha,\beta\} = \{1,\beta^*\}$ is also the global maximum. □

A direct implication of Proposition 4 is that providing the committee with expert information is beneficial as long as the agents play the mixed strategy equilibrium:

Corollary 4. *The mixed strategy equilibrium identified in Proposition 3 outperforms individually informative voting and obedient voting.*

The corollary holds because individually informative voting is equivalent to $\alpha = \beta = 1$ and obedient voting $\alpha = \beta = 0$, and Proposition 4 has just shown that the mixed strategy equilibrium ($\alpha^* = 1$ and $\beta^* \in (0,1)$) is optimal with respect to the choice of α and β.

Recall that the mixed strategy equilibrium does not exist when the accuracy of expert information q is above the threshold $\bar{q}$. In light of Proposition 4, this means that when q is very high it is not worthwhile to combine both types of information to maximize $P(\alpha,\beta)$. The following remark gives us some indication about the threshold.

Remark 2. $\bar{q} > P_C$, that is, the upper bound of the accuracy of expert information for the mixed strategy equilibrium to exist is higher than the accuracy of sincere voting without expert information.

Proof. Let $\beta^*(\cdot)$ be the equilibrium/optimal β as a function of q, and let $q_C \equiv P_C$. By construction we have $\beta^*(\bar{q}) = 0$ and $\beta^*(q_C) \geq 0$. In fact we have $\beta^*(q_C) > 0$. To see this, note $\beta^*(q_C) = 0$ implies $P_H(q_C) = q_C$, which is a contradiction since i) $q_C > p$; ii) from (16) if $q = p$ then $\beta^* = 0$ and thus $P_H(p) = q_C$; and iii) by differentiating (17) with respect to q we see that $P_H(q)$ is strictly increasing.

From (15) we can check that $\beta^*(q)$ is strictly decreasing in q. Thus $\beta^*(q_C) > \beta^*(\bar{q}) = 0$ implies $\bar{q} > q_C = P_C$. □

In view of Proposition 4, the remark implies that for private signals to be optimally disregarded for efficiency maximization, the public signal has to be strictly better than the efficiency the private signals can achieve collectively through majority voting.

4 Discussion and Conclusion

This paper has studied the effects of a public signal in a model of majority decision making with common interests. We have demonstrated that the presence of public signal changes the structure of voting equilibria substantially. In particular, individually informative voting is not an equilibrium when the precision of the public signal is better than each agent's individual signal. If the expert information is not too accurate, there are two informative equilibria, namely the obedient equilibrium and the mixed strategy equilibrium. The latter not only outperforms the former but also is efficient in the sense that the mixing probabilities coincide with those chosen to maximize the accuracy of the majority decision. A fortiori, it outperforms the sincere voting equilibrium in the absence of public signal. If the expert information is very accurate, then the only informative equilibrium involves obedient voting, whereby every agent follows expert information, and this equilibrium is indeed efficient.

Our analysis indicates that as long as the committee members can coordinate to play the efficient equilibrium, expert information to be observed by all members of a committee is welfare enhancing. However, there remains an empirical question whether it is realistic to expect them to play the highly sophisticated mixed strategy equilibrium, since as we saw above the computation of the equilibrium mixed strategy is complex. Furthermore, our model has a much simpler informative equilibrium (the obedient equilibrium) but the efficiency in the equilibrium (which is exactly the accuracy of the expert information) can be lower than the sincere voting equilibrium in the absence of public information, where the standard CJT holds. This suggests that, if agents play the obedient equilibrium, it may be that expert information has "excessive" influence on the voting outcome and the committee is better off without expert information.

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